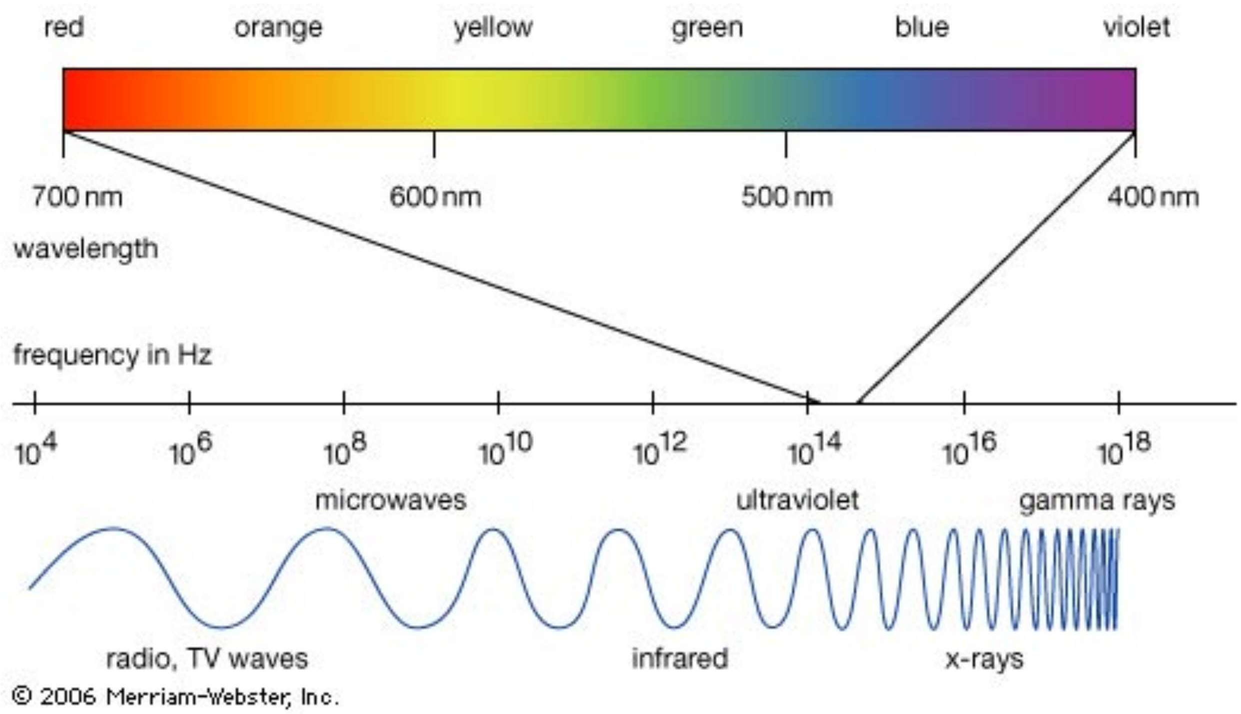
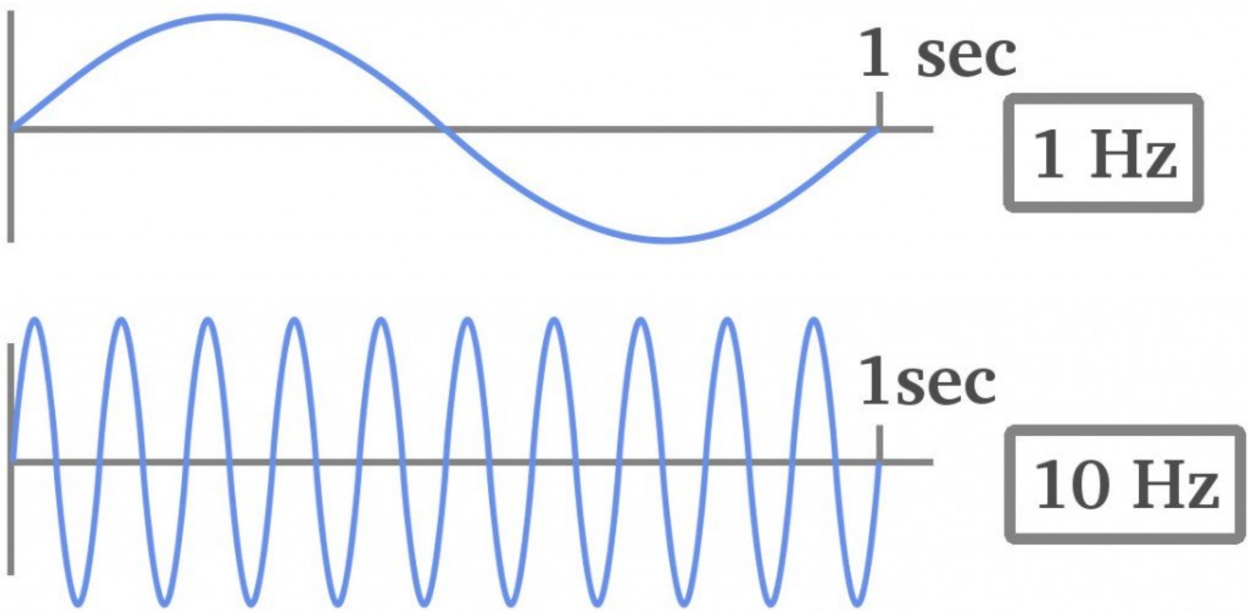
Chapter 7, Lesson 7 Part II

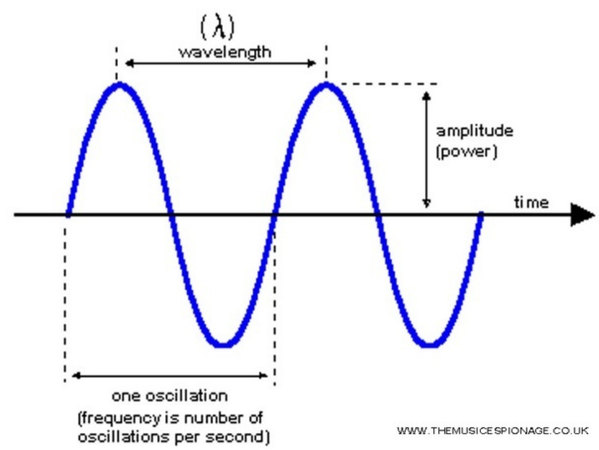
Problem 3 is about radio waves. Sound and light travel in waves. You can see how fast waves travel by how squished it looks in the diagram. Gamma rays are the fastest while radio waves are the slowest.



Each kind of wave has its own frequencies. The color red has a slower frequency than the color violet. Gamma rays from space are waves that cycle even faster than the ultraviolet waves that we see which means they have higher frequency than UV light. Infrared waves used in devices that help you see at night are slightly slower than light waves. Microwaves cycle slower and radio and television waves move even slower.

You may be wondering what the frequency is. Frequency is the number of cycles that a wave makes per cycle. When it comes to sound, higher pitches have higher frequencies. Every time a note in music jumps an octave, the frequency doubles. The diagram below shows the difference between a frequency of 1 Hz and 10 Hz. You may want to watch [this video for a visual demonstration of wave terms](#).

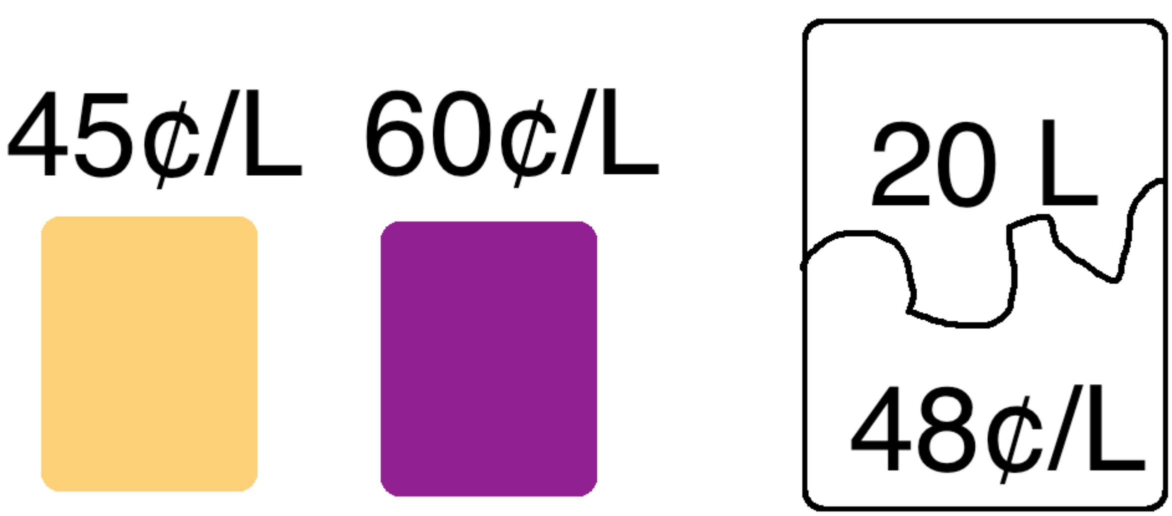




The wavelength is the distance from crest to crest (the top of the wave) or from trough to trough (the bottom of the wave).

In Problem 3, you are given a formula for the wavelength *w* using frequency *f* as the input. Hopefully, this problem can help you see how math and science work hand in hand.

A classic mixture problem involves making a blend of two different types of ingredients. Suppose you are making a blend of apple juice which is 45 cents a liter and cranberry juice is 60 cents a liter. The goal is to make 20 liters of a juice blend that costs 48 cents per liter.



When writing equations, focus on units. The goal is to make 20 liters, so define the variables based on liters.

a = liters of apple juice

c = liters of cranberry juice

Since we are mixing two kinds of juices together, what mathematical operation is that? Addition. What is the total amount of juice? 20 liters

$$a + c = 20$$

The other goal is the cost of the blend. If you make 20 liters of juice that costs 48 cents per liter, how much does it cost? A little analysis can help if you are unsure of what to do.

Look at the following pattern:

1 L blend = 48¢
2 L blend = 96¢
10 L blend = 480¢
20 L blend = 960¢

Apply this pattern to the juices.

1 L apple = 45¢
2 L apple = 90¢
10 L apple = 450¢
a L apple = 45a¢

1 L cranberry = 60¢
2 L cranberry = 120¢
10 L cranberry = 600¢
c L cranberry = 60c¢

Just like you added the volume (liters), you can add the cost (cents).

$$45a + 60c = 960$$

The cost equation has a lot of big numbers. It is good to pause and see if a number can be divided out. All the terms have a 0 or 5 in the ones place so they are all divisible by 5. All terms are divisible by 3 because $4 + 5 = 9$, $6 + 0 = 6$, and $9 + 6 + 0 = 15$. This is why divisibility rules are so important so brush up on your Pascal's triangles! If all terms are divisible by 3 and 5, they are also divisible by 15 which is the divisor.

$$\frac{45a + 60c = 960}{15}$$

$$3a + 4c = 64$$

You can solve the simultaneous equations by any method. I am going to multiply the first equation by 3 and subtract it from the second.

$$\begin{array}{r} 3a + 4c = 64 \\ \underline{-3a - 3c = -60} \end{array}$$

$$c = 4$$

Solve for a.

$$a + 4 = 20$$

$$a = 16$$

We do not know if your answer is correct until we plug it into the other equation as a check.

$$3a + 4c = 64$$

$$3 \cdot 16 + 4 \cdot 4 = 64$$

$$48 + 16 = 64$$

Word problems need another check. Does it make sense? If the blend is cheaper, it needs more of the cheaper juice. The amount of apple juice (16 liters) is much greater than the amount of cranberry juice (4 liters). It makes sense!

The final blend problem is about shopping. A friend busts out of quarantine and buys 95 plants, spending $353 in one shot. Some of the plants cost $3 each and the others cost $5 each. You have enough information to figure out how many of each kind of plant were bought.

First, define variables:

x = number of cheaper plants ($3)

y = number of pricey plants (\$5)

The total number of plants is 95 so the first equation is about the number of plants. Almost every first equation takes this form.

$$x + y = 95$$

The second equations focuses on the cost. How much do the cheaper plants cost? There are x plants that cost \$3 each: $3x$. How much do the cheaper plants cost? There are y plants that cost \$5 each: $5y$. What is the total cost? \$353

$$3x + 5y = 353$$

There is no way to simplify the equation because the terms have nothing in common. You can solve this equation by substitution.

$x + y = 95$

$x = 95 - y$

$3x + 5y = 353$

$3(95 - y) + 5y = 353$

$285 - 3y + 5y = 353$

$$\begin{array}{rcl} 285 + 2y & = & 353 \\ \underline{-285} & & \underline{-285} \end{array}$$

$2y = 68$

$y = 34$

You are not finished because you have not solved for x.

$x = 95 - y$

$x = 95 - 34$

$x = 61$

You are not finished because you have not checked.

$3x + 5y = 353$

$3 \cdot 61 + 5 \cdot 34 = 353$

$183 + 170 = 353$

The solution (61,34) makes sense. The numbers are positive integers that are less than 95.

Homework

3. The wavelength of a radio wave is a function of its frequency. A formula for this function is

$$w = \frac{300{,}000}{f}$$

in which w represents the wavelength in meters and f represents the frequency in kilocycles per second.

a) What kind of variation is this?
b) What happens to the wavelength as the frequency of a radio wave increases?
c) Solve the formula for f.
d) Find the frequency of a radio wave whose wavelength is 2,000 meters.

8. A test contains 42 questions, of which some are worth 2 points and the rest are worth 3 points. A perfect score is 100 points. Find out how many questions of each type are on the test by doing each of the following.

a) Letting x and y represent the numbers of questions worth 2 and 3 points respectively, write an equation relating x, y, and 42.
b) Express the total number of points possible from the 2-point questions in terms of x.
c) Express the total number of points possible from the 3-point questions in terms of y.
d) Write an equation expressing the fact that the total number of points possible on the test is 100.
e) Solve the simultaneous equations that you have written for x and y.
f) How many questions worth 2 points and how many questions worth 3 points are on the test?

9. Acute Alice works Saturdays in a nut shop. She is supposed to add some Spanish peanuts worth 84 cents a pound to 40 pounds of Virginia peanuts worth 71 cents a pound to make a mixture worth 79 cents a pound. Find out how many pounds of Spanish peanuts she should add by doing each of the following.

a) Letting x represent the number of pounds of Spanish peanuts added and y the number of pounds of peanuts in the mixture, write an equation relating x, y, and 40.
b) In terms of x, how much are the Spanish peanuts worth?
c) In terms of y, how much is the mixture worth?
d) Write an equation relating the worth of the two kinds of peanuts used to the worth of the mixture.
e) Solve the simultaneous equations that you have written for x and y.
f) How many pounds of Spanish peanuts should Alice use?

10. One evening, 1,255 people went to the Orpheum Theatre to see *Gone with the Wind*. The box office receipts totaled $3,680, the price of admission for adults being $3 and that for children being $2. Find out how many tickets of each type the theater sold by doing each of the following.

a) Letting x represent the number of adult tickets sold and y the number of children's tickets, write a pair of simultaneous equations, one relating the numbers of tickets and the other relating their costs.
b) Solve the equations.
c) How many tickets of each type did the theater sell?