

Chapter 7, Lesson 7 Part I

Suppose I told you I have 20 coins, all dimes and nickels, that added up to \$1.30. Could you figure out how many dimes and how many nickels I have?

Yes. You could get 20 dimes and 20 nickels and do trial and error. Eventually, you would get the answer. You might get lucky and guess right the first time. It might take longer.

You can also solve it algebraically. The first step is to define your two variables, d and n .

d = the number of dimes

n = the number of nickels

Now you can write an equation. The number of coins is 20. So, the number of dimes and nickels is 20:

$$d + n = 20$$

You cannot solve this problem until you write a second equation. What is the other piece of information? I have \$1.30. Since decimals are not everyone's strengths, we need to write the amount of cents. How many pennies are in \$1.30? There are 130 pennies or 130 cents in \$1.30.

Suppose all the coins were dimes. How would you calculate the amount of cents? There are 10 cents per dime, so 20 dimes is 200 cents. If we don't

know the number of dimes, we could write $10d$ for the value of dimes.

Suppose all the coins were nickels. How would you calculate the amount of cents? There are 5 cents per nickel, so 20 nickels is 100 cents. If we don't know the number of nickels, we could write $5n$ for the value of nickels.

Putting the three together, we can make our second equation:

$$10d + 5n = 130$$

You can solve the two simultaneous questions by graphing or by the addition/subtraction/multiplication method or by substitution.

Let's try substitution.

$$d + n = 20 \text{ so } d = 20 - n$$

$$10d + 5n = 130$$

$$10(20 - n) + 5n = 130$$

$$200 - 10n + 5n = 130$$

$$200 - 5n = 130$$

$$\begin{array}{r} -200 \\ -200 \end{array}$$

$$\frac{-5n = -70}{-5 \quad -5}$$

$$n = 14$$

$$d + n = 20$$

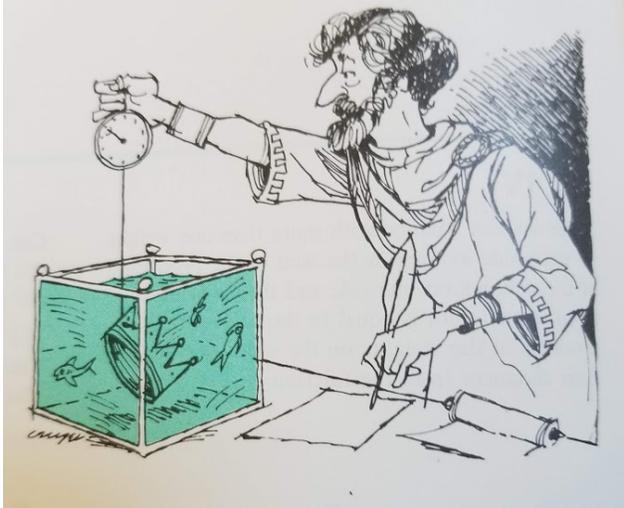
$$d + 14 = 20$$

$$d = 6$$

To check, 6 dimes is 60 cents and 14 nickels is 70 cents. Add that together and you get \$1.30.

This is a mixture problem because you have a mixture of coins. There is a [very ancient mixture problem about Archimedes](#). We are going to solve it using simultaneous equations.

To build a math model, one must make an assumption. We are going to assume the crown the king was given was made of gold and silver. It could have been made of gold and a different metal. The idea is to build a model for one situation and apply it in other ways. This, too, is a mixture problem because we are looking at a mixture of metals.



Archimedes weighed the crown. Suppose it weighed 2 kg. He did not use metric units, but we can.

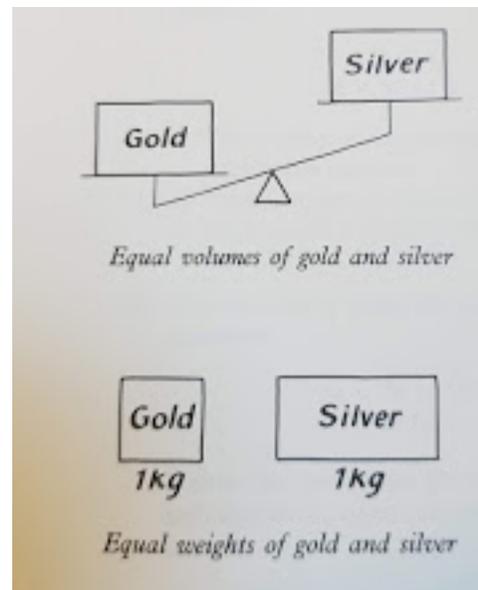
What is the weight of gold? If the merchant was honest, it would be 2 kg. Since we do not know the weight, suppose x is the weight of gold in kg. In case the merchant cheated, we also need to come up with the weight of silver. Suppose y is the weight of silver.

In most mixture problems, you have an equation that adds two variables. In the last one, we added dimes and nickels. In this one, we add the weight of gold x to the weight of

silver y to make the crown's weight, 2 kg. This kind of equation is going to look the same in most problems.

$$x + y = 2$$

The second equation has to do with the volume of each metal. The volume is how much space the metals take up. This is why Archimedes dunked the crown in water. Gold is more dense than silver. If you had a cube of gold and a cube of silver that were the same size, the gold cube would be heavier.



If you pour 1 kg of liquid gold into a box and 1 kg of liquid silver into

another box, the gold would take up less space. It would fill a smaller box. A pure gold crown would take up less space than a crown made of a mixture of gold and silver.

Archimedes did not know the density of metals so he had to use water. We do, so we can do some math. One kilogram of gold fills up 50 cubic centimeters with the pattern:

$$1 \text{ kg gold} = 50 \text{ cc}$$

$$2 \text{ kg gold} = 100 \text{ cc}$$

$$3 \text{ kg gold} = 150 \text{ cc}$$

$$4 \text{ kg gold} = 200 \text{ cc}$$

$$x \text{ kg gold} = 50x \text{ cc}$$

One kilogram of silver fills up 100 cubic centimeters so y kilograms of silver fills up $100y$ cubic centimeters.

Suppose the volume of the crown is 140 kilograms. We can add the two volumes to make a second equation.

$$50x + 100y = 140$$

To simplify the equation, divide both sides by 10.

$$5x + 10y = 14$$

Multiply the weight equation by -2 and apply the addition method.

$$\begin{array}{r} 5x + 10y = 14 \\ \underline{-5(x + y = 2)} \end{array}$$

$$\begin{array}{r} 5x + 10y = 14 \\ \underline{-5x - 5y = -10} \end{array}$$

$$\frac{5y}{5} = \frac{4}{5}$$

$$y = \frac{4}{5} = 0.8$$

$$x + y = 2$$

$$\begin{array}{r} x + 0.8 = 2.0 \\ \underline{-0.8} \quad \underline{-0.8} \end{array}$$

$$x = 1.2$$

To check, plug in the second equation.

$$50x + 100y = 140$$

$$50(1.2) + 100(0.8) = 140$$

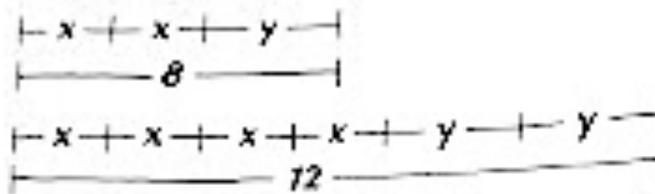
$$60 + 80 = 140$$

$$140 = 140$$

The merchant is a cheat because the crown has 1.2 kilograms of gold and 0.8 kilograms of silver.

Homework

1. This diagram illustrates a pair of simultaneous equations.



- What are the equations?
 - How many solutions does the pair of equations have?
 - What single word describes such a pair of equations?
 - What would the graph of this pair of equations consist of?
2. Write an equation that, together with the equation

$$x + 2y = 5,$$

makes a pair of simultaneous equations

- that are inconsistent,
- that are equivalent.
- that have the solution $(1, 2)$.

4. A telephone coin box contains 52 coins, of which some are nickels and the rest are dimes. The total value of the coins is \$4.50. Find out how many coins of each type it contains by doing each of the following.

- Letting x represent the number of nickels and y represent the number of dimes, write an equation relating x , y , and 52.
- Express the total value of the nickels in cents in terms of x .
- Express the total value of the dimes in cents in terms of y .
- Write an equation expressing the fact that the total value of the coins is 450 cents.

- Solve the simultaneous equations that you have written for x and y .
- How many nickels and how many dimes does the coin box contain?

7. Christmas tree tinsel is a mixture of lead and tin. Ten cubic centimeters of it weighs 83.4 grams. One cubic centimeter of tin weighs 7.2 grams and one cubic centimeter of lead weighs 11.0 grams. Find out how many grams of each metal are in ten cubic centimeters of tinsel by doing the following.

- Letting x and y represent the numbers of cubic centimeters of tin and lead respectively, write a pair of simultaneous equations, one relating volumes (numbers of cubic centimeters) and the other relating weights.
- Solve the equations.
- How many *grams* of each metal are in ten cubic centimeters of tinsel?