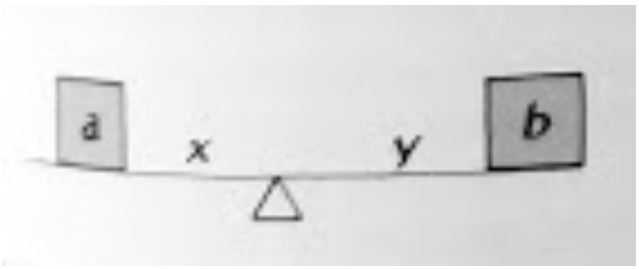
Chapter 7, Lesson 6

Hopefully, you had fun with the warm-up activity I spent last night coding just for you! The take away is that if two people have different weights, the lighter one must be closer to the fulcrum and the heavier one must be farther away from the fulcrum.

This picture shows person who weighs a pounds and a person who weighs b pounds on a seesaw. The a-pound person is x feet from the fulcrum; the b -pound person is y feet from it.



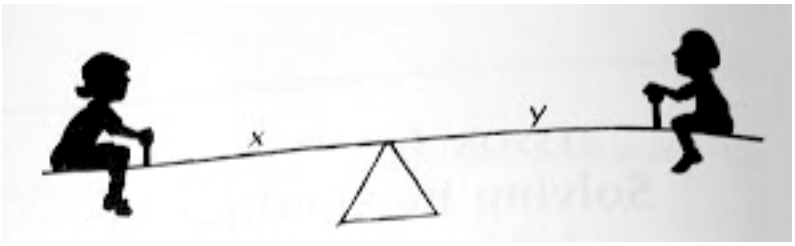
When you were a child, you and your friends figured out how to sit and stay in balance. There is a rule for doing so mathematically.

$$ax = by$$

If we divide both sides by b, we now see the equation in what form?

$$y = \frac{ax}{b}$$

What kind of function is it (direct variation, inverse variation, or linear)?



Suppose a 40-pound girl is at distance x from the fulcrum and the 20-pound boy is at distance y. We can plug this into the rule.

$$40x = 20y$$

To solve for y, we divide both sides by what number?

$$y = 2x$$

These kids decide to keep a social distance of 12 feet. How can we apply this to x and y?

$x + y = 12$

We now have two equations. One way to solve is graphically. To solve by writing equations, you would have to rearrange an equation and that is too much of a hassle. There is an easier method which you will apply a lot in geometry next year. It is called the substitution method.

Plug $y = 2x$ into $x + y = 12$. Since y equals $2x$, you can replace the y with $2x$ in the second equation.

$x + 2x = 12$

$$\frac{3x = 12}{3 \quad 3}$$

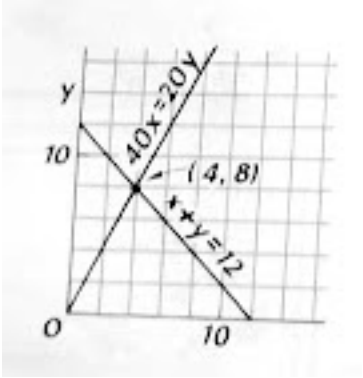
$x = 4$

Finding y is easy peasy.

$y = 2x$

$y = 2 \cdot 4$

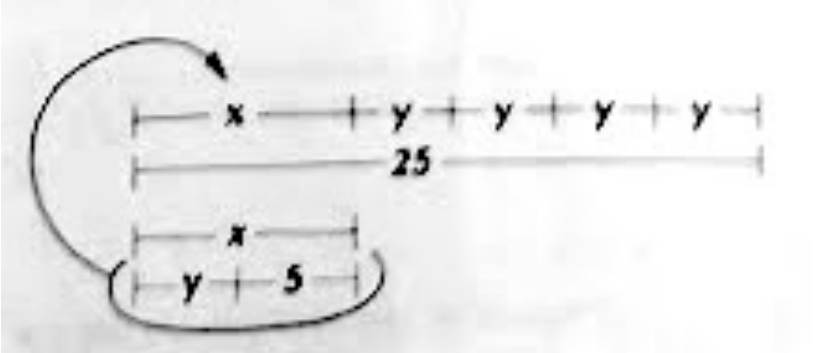
$y = 8$



Checking is easy.

$4 + 8 = 12$

The diagram in this example illustrates the substitution method. Since $x = y + 5$, you can plug $y + 5$ for every x in the top equation.



$$x + 4y = 25$$

$$y + 5 + 4y = 25$$

$$5y + 5 = 25$$
$$-5 \quad -5$$

$$\frac{5y = 20}{5 \quad 5}$$

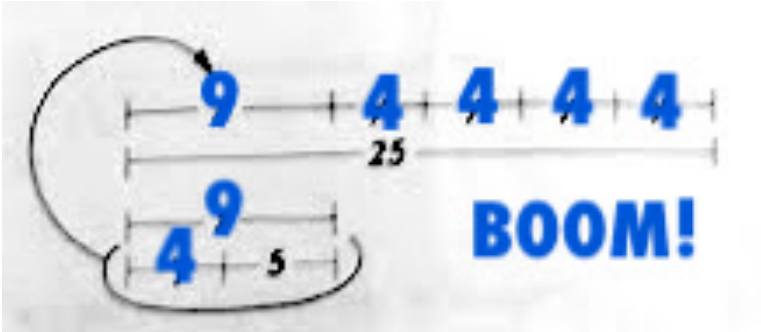
$$y = 4$$

$$x = y + 5$$

$$x = 4 + 5$$

$x = 9$ so the solution is (9,4).

The diagram is easy to check.



Another example of solving by substitution is below.

$$2x + 3y = 8$$
$$y - x = 11$$
$$+x \qquad +x$$

$$y = x + 11$$

Now plug into the first equation.

$$2x + 3(x + 11) = 8$$

$$2x + 3x + 33 = 8$$

$$5x + 33 = 8$$
$$-33 \quad -33$$

$$\frac{5x = -25}{5 \qquad 5}$$

$$x = -5$$

$y - (-5) = 11$

$y + 5 = 11$

$y = 6$

Plug into the OTHER equation to check.

$2x + 3y = 8$

$2(-5) + 3(6) = 8$

$-10 + 18 = 8$

THE MOST IMPORTANT THING TO REMEMBER IN THE HOMEWORK: if the directions say to solve by substitution, then solve by substitution. If you choose another method, it is not correct. This is not the time for freestyle!

HOMEWORK

Set I

1. Use a number line to find each of the following.
   a) The distance between −5 and 6.
   b) The number of the point midway between −9 and 1.
   c) The numbers corresponding to the two points whose distance from 8 is 2.
   d) The numbers corresponding to the two points whose distance from −2 is 7.

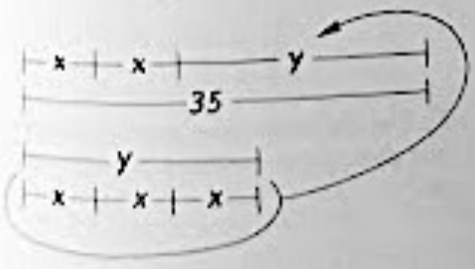
2. The following questions are about the axes of a coordinate graph.
   a) What is the equation of the x-axis?
   b) What is its slope?
   c) What is the equation of the y-axis?
   d) What is its slope?

3. In a contest in 1973, a modified Opel station wagon got 376 miles per gallon!
   a) Write a formula for the distance in miles, d, that this car could go on g gallons of gasoline if it continued to get this mileage.
   b) Solve the formula for g in terms of d.
   c) Use your formula for part b to find the amount of gasoline used by the car to travel the 14 miles in the contest. (Round your answer to the nearest hundredth.)

Set II

The diagrams for exercises 4 through 7 represent pairs of simultaneous equations. Write the equations and solve them by making the indicated substitution to find the lengths of the line segments in each diagram.

4.



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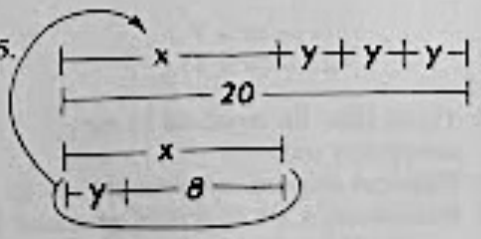

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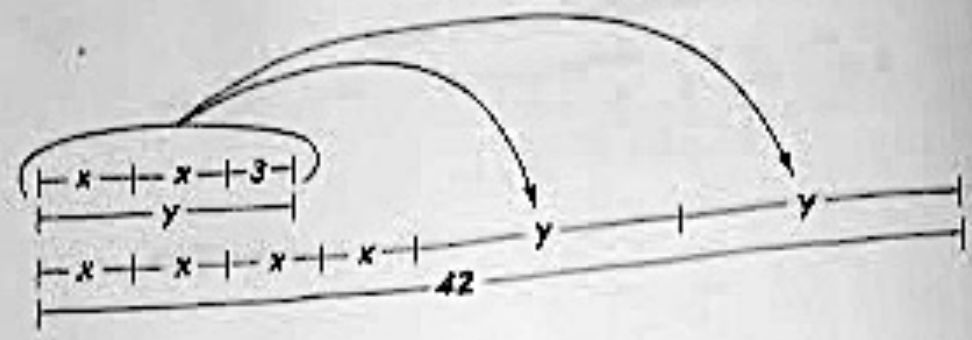

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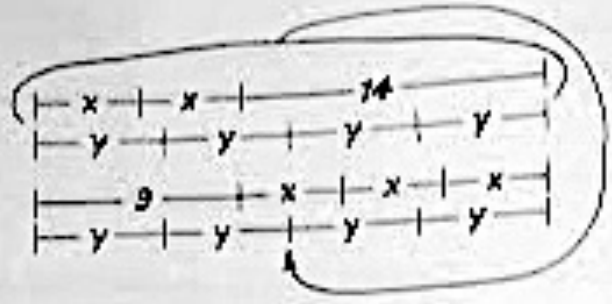


Solve the following simultaneous equations by the substitution method. Show your steps and check your answers.

8. $x = y$
   $2x = y + 8$

9. $y = x - 2$
   $3(x + 1) = 4y$

10. $y = 7x + 10$
    $y = 4 + x$

11. $y = 3x - 1$
    $x + 2y = 33$

12. This exercise is about the simultaneous equations

$$x = 3y + 7$$
$$2x - 6y = 11$$

a) Try to solve the equations by the substitution method.
b) What can you conclude from the result?
c) What would a graph of these equations consist of?

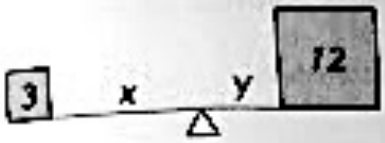
13. This exercise is about the simultaneous equations

$$4y - 8x = 4$$
$$y = 2x + 1$$

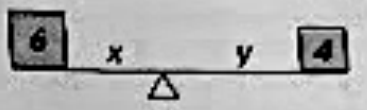
Follow the instruction given in exercise 12 and answer the questions asked in parts b and c.

The diagrams for exercises 14 through 16 represent weights balanced on seesaws. For each exercise, do each of the following:
a) Use the clue and the diagram to write a pair of simultaneous equations.
b) Solve the equations to find x and y.
c) Check your solution to see if it makes the seesaw balance.

14. The distance between the weights is 35.



15. The distance between the weights is 20.



16. The sum of the weights is 24.